Available online at www.sciencedirect.com
ScienceDirect
Advances in Space Research 62 (2018) 65–70
ADVANCES IN SPACE RESEARCH
(a COSPAR publication)
www.elsevier.com/locate/asr

Problem of stability of multidimensional solutions of the BK class equations in space plasma

Vasily Yu. Belashov [a,*], Elena S. Belashova [b], Oleg A. Kharshiladze [c]

[a] Kazan Federal University, Kazan, Russia
[b] Kazan National Research Technical University named after A.N. Tupolev – KAI, Kazan, Russia
[c] Iv. Javaxishvilis Tbilisi State University, Tbilisi, Georgia

Received 22 November 2017; received in revised form 31 March 2018; accepted 3 April 2018
Available online 12 April 2018

Abstract

The problem of stability of the multidimensional solutions of the BK class equations describing the nonlinear waves which are forming on the low-frequency branch of oscillations in plasma for cases when $\beta \equiv 4\pi nT/B^2 \ll 1$ and $\beta > 1$ is studied. In first case, for $\omega < \omega_B = eB/Mc, k\lambda_D \ll 1$ the FMS waves are excited, and their dynamics under conditions $k_x^2 \gg k_\perp^2,\ v_x \ll c_A$ near the cone of $\theta = \arctan\,(M/m)^{1/2}$, is described by the equation of the BK class known as the GKP equation for magnetic field $h = B_\sim/B$ with due account of the high order dispersive correction defined by values of plasma parameters and angle $\theta = (\mathbf{B}, \mathbf{k})$. In another case, the dynamics of the finite-amplitude Alfvén waves propagating near-to-parallel to $\mathbf{B}$ is described by the equation of the same class known as the 3-DNLS equation for $h = (B_y + iB_z)/2B|1-\beta|$. To study the stability of multidimensional solutions in both cases the method of investigation of the Hamiltonian bounding with deformation conserving momentum by solving the variation problem is used. As a result, we have obtained the conditions of existence of the 2D and 3D soliton solutions in the BK system for cases of the GKP and 3-DNLS equation (i.e. for the FMS and Alfvén waves, respectively) in dependence on the equations' coefficients, i.e. on the parameters of both plasma and wave.

Keywords: Nonlinear waves; Stability; BK class equations; GKP equation; 3-DNLS equation; FMS waves; Alfvén waves; Space plasma

1. Introduction. Basic equations

In this paper we study the formation, structure, stability and dynamics of the multidimensional solitons forming on the low-frequency branch of oscillations in a plasma for cases $\beta \equiv 4\pi nT/B^2 \ll 1$ and $\beta > 1$. These oscillations are described by the Belashov-Karpman (BK) class of equations

$$\partial_t u + \mathrm{A}(t,u)u = f, \quad f = \kappa \int_{-\infty}^{x} \Delta_\perp u dx,$$
$$\Delta_\perp = \partial_y^2 + \partial_z^2 \tag{1}$$

which with operator

$$\mathrm{A}(t,u) = \alpha u \partial_x - \partial_x^2(\nu - \beta\partial_x - \gamma\partial_x^3) \tag{2}$$

turns into the generalized Kadomtsev-Petviashvili (GKP) equation and in case when $\beta \equiv 4\pi nT/B^2 \ll 1$ for $\omega < \omega_B = eB/Mc,\ k\lambda_D \ll 1$ describes propagation of the fast magnetosonic (FMS) waves in magnetized plasma with $k_x^2 \gg k_\perp^2,\ v_x \ll c_A$ near the cone of $\theta = arctan\ (M/m)^{1/2}$ (Belashov, 1994). In this case function u has a sense the dimensionless amplitude of the magnetic field of the wave,

* Corresponding author at: Institute of Physics, Kazan Federal University, 18 Kremlyovskaya, Kazan 420008, Russia.
E-mail address: vybelashov@yahoo.com (V.Yu. Belashov).

https://doi.org/10.1016/j.asr.2018.04.001
0273-1177/